The six equations resulting from the computation of $(\partial F/\partial m_{ij})$, with $i = 1,2$ and $j = 1,2,3$, are

$$2\lambda_a m_{11} + \lambda_c m_{21} = -k_{11} \quad (5)$$

$$2\lambda_a m_{12} + \lambda_c m_{22} = -k_{21} \quad (6)$$

$$2\lambda_a m_{13} + \lambda_c m_{23} = 0 \quad (7)$$

$$\lambda_c m_{11} + 2\lambda_b m_{21} = -k_{12} \quad (8)$$

$$\lambda_c m_{12} + 2\lambda_b m_{22} = -k_{22} \quad (9)$$

$$\lambda_c m_{13} + 2\lambda_b m_{23} = 0 \quad (10)$$

It is not very convenient to solve the nine equations given from Eqs. (3–10) in the unknowns λ_a, λ_b, λ_c and m_{ij}. The attention can merely be focused on the value of m_{13} and m_{23}. Consider therefore the 6×6 determinant, built by the coefficients of the m_{ij} in Eqs. (5–10). Its value is obviously equal to $(4\lambda_a\lambda_b - \lambda_c^2)$,[3] and one may denote it shortly by d. As long as $d \neq 0$, it can easily be checked that $m_{13} = m_{23} = 0$ always belongs to the solution.

If d is zero, one can have $\lambda_a = \lambda_b = \lambda_c = 0$, which yields an undefined system where V_1 and V_2, or W_1 and W_2, or V_i and W_j $(i \neq j)$ are both zero vectors. Let one assume that all multipliers are different from zero and $d = 0$. In that case, the Eqs. (5) and (8), as well as Eqs. (6) and (9) become dependent. In order to be compatible, these equations yield the following condition:

$$2\lambda_a/\lambda_c = \lambda_c/2\lambda_b = k_{11}/k_{12} = k_{21}/k_{22}$$

or

$$k_{11}k_{22} - k_{12}k_{21} = 0 \quad (11)$$

If one substitutes the separate elements of PA and QB in Eq. (11), one can easily check that the condition Eq. (11) is only fulfilled if A or B contains a row of zeros, i.e., a vector V_i or W_i is a null vector. Hence, d is never zero within the scope of the problem one has to solve.

From the previous considerations, it follows that one has $m_{13} = m_{23} = m_{31} = m_{32} = 0$ and $m_{33} = \pm 1$. If $m_{33} = -1$, it is always possible to rewrite the solution for M as

$$M_- = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ \sin\varphi & -\cos\varphi & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

whereas for $m_{33} = 1$, one can state M as follows:

$$M_+ = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The angle φ has to be determined from Eq. (2). To simplify the notations, one introduces the new symbols

$$a_+ = k_{11} + k_{22};\ b_+ = k_{21} - k_{12};\ c_+ = (a_+ + b_+)^{1/2} \quad (12)$$

$$a_- = k_{11} - k_{22};\ b_- = k_{21} + k_{12};\ c_- = (a_- + b_-)^{1/2} \quad (13)$$

Now it becomes obvious that in each assumption for m_{33}, one has

$$\max_{\varphi_-} F_0(M_-) = C_-;\ \max_{\varphi_+} F_0(M_+) = C_+$$

by setting

$$\cos\varphi_\pm = a_\pm/c_\pm;\ \sin\varphi_\pm = b_\pm/c_\pm \quad (14)$$

By an explicit development of C_+ and C_-, one obtains $C_- < C_+$ because

$$a_+^2 + b_+^2 > a_-^2 + b_-^2$$

or

$$k_{11}k_{22} - k_{12}k_{21} > 0$$

which can be computed by substituting the elements of PV_i and QW_i in it. This yields

$$k_{11}k_{22} - k_{12}k_{21} = (PV_1 \mathbf{X} PV_2)_z (QW_1 \mathbf{X} QW_2)_z$$

which is always positive by the previous definition of P and Q. The solution is thus $R = P^T M(\varphi_+) Q$.

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A Resonance Igniter for Hydrogen-Oxygen Combustors

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ONE of the problems in using H_2/O_2 as propellants for a rocket, is that of devising a simple yet reliable method for multiple ignitions. Commonly used methods are high-energy spark igniters or the injection of fluorine which is hypergolic with the fuel. Both systems, however, are complex and, in the case of fluorine, there are toxicity and compatibility complications. An ignition technique that avoids these problems is based on the resonance heating,[1–3] a phenomenon which can be readily demonstrated. A hole of $\sim\frac{1}{4}$ in. diam with a depth of 1 to 3 in. is drilled into a block of wood. Air is supplied at 125 psia to a simple $\frac{1}{16}$-in.-diam choked nozzle and the nozzle is pointed into the hole. By varying the gap between the nozzle and the hole opening, a position can be found which results in the resonant coupling of the shocks in the jet with the dynamics of the gas in the hole. Intense sound is radiated, and the wood in the bottom of the hole is charred. The generally accepted model for the heating assumes that the underexpanded jet excites a periodic shock wave which oscillates up and down inside the hole, heating the gas which is trapped within the hole.

Attempts to predict the heating have been only moderately successful due to the complexity of the phenomenon. The most elaborate analysis known by the authors is that of Kang,[2] in which it was predicted that, with a perfectly insulated hole, the absolute gas temperature could be increased tenfold.

It was envisioned that a suitable igniter would be a simple choked nozzle fed by gaseous O_2 and H_2 with the nozzle mounted a fixed distance from an insulated hole (see Fig. 1). To facilitate fabrication and decrease the scope of the experiment, the following variables were arbitrarily fixed for initial testing: gas total pressure—80 psia; ambient pressure in the region between nozzle and hole—14.4 psia; nozzle diameter—0.200 in., nozzle shape—simple, choked, circular; hole diameter—0.280 in.; hole material—brass; hole-nozzle alinement—coaxial; and gas composition—stoichiometric mixture of H_2 and O_2.

The first apparatus tried consisted of a brass cylinder of 0.75-in. O.D. with an 0.25-in.-diam hole. An adjustable pis-

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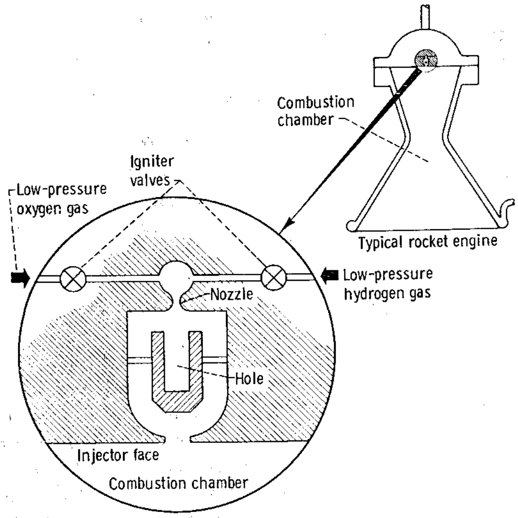


Fig. 1 Schematic drawing of resonance-igniter concept in rocket-engine application.

ton with an O-ring seal was inserted into the hole to vary its depth. Gas temperature was measured by an insulated iron-constantan thermocouple attached to the movable piston. The nozzle diameter of 0.200 in. was chosen based on the results of Ref. 1 that indicated a nozzle diameter slightly smaller than the hole diameter would give high temperatures. The stoichiometry of the H_2-O_2 mixture was defined by the relative flow rates of the gases as measured by orifice meters upstream of the nozzle where the gases were mixed. The experimental variables were nozzle-hole gap and hole depth. The initial tests revealed maxima in the measured temperature as functions of both nozzle gap and hole depth although the maximum temperature was insufficient to cause ignition due to the heat leak to the massive brass cylinder (Fig. 2).

The variation in temperature as a function of nozzle gap is due to the spatially periodic nature of the shocks in the under-expanded jet. In order for the oscillations in the hole to be excited, it is necessary for one of the spatially periodic standing shocks to be near the hole entrance.[1] Hence, moving the nozzle relative to the hole results in a great variation in the excitation of the oscillations with a peak in the oscillation amplitude whenever the shock is near the entrance to the hole.

The variation in temperature as a function of the hole depth is not so readily explained. According to Ref. 1, wherein a single peak was recorded, the maximum in temperature as a function of hole depth was attributed to an increase in temperature caused by a steepening of the shock as

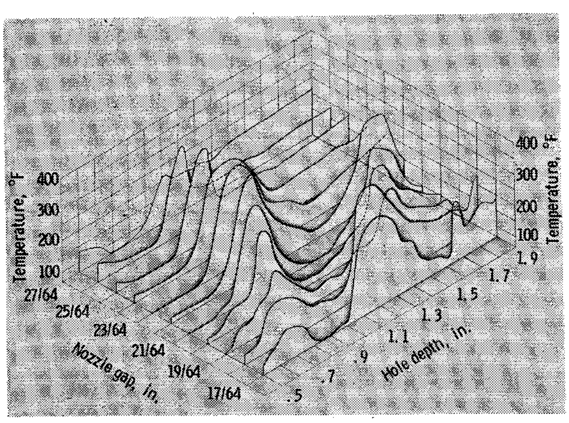


Fig. 2 Effect of nozzle gap and hole depth on gas temperature at bottom of resonator cavity. Working fluid stoichiometric mixture of hydrogen and oxygen gases at 70°F (294°K).

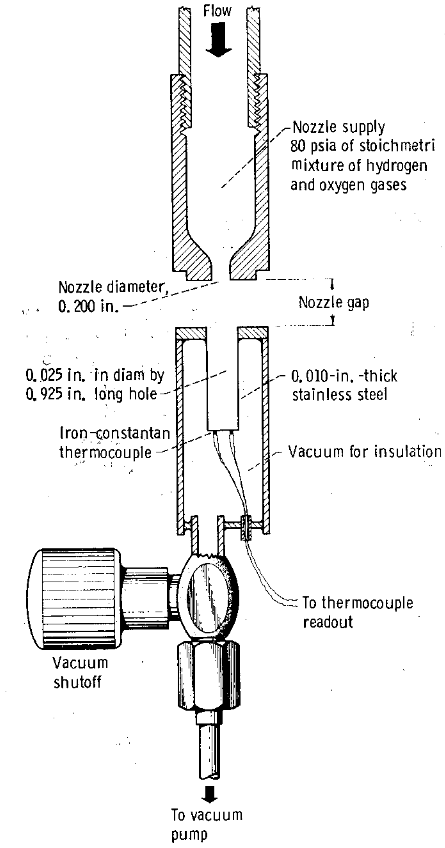


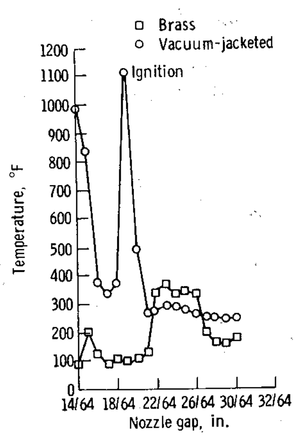
Fig. 3 Vacuum jacketed hole.

the hole depth was increased and a corresponding simultaneous decrease in temperature due to the increased heat leak as the hole depth was increased. The explanation does not seem to cover the present case where two maxima are observed, however.

Another possible explanation is the excitation of a second harmonic mode in the hole which would oscillate at the same resonant frequency as the initial resonance but at twice the length. The phenomena of modal shifts may serve to explain the series of peaks in temperature as hole depth is increased, but additional experimentation involving the measurement of the frequency of oscillation would be required to substantiate any explanation.

In the second phase of the experimentation, a fixed-depth vacuum-jacketed configuration was constructed. The hole depth was chosen to correspond to one of the peaks in temperature obtained with the all-brass hole, and the vacuum jacketing was included to minimize the heat leak. A view of the modified experimental apparatus is shown in Fig. 3. The new configuration was tested with the same nozzle and at the

Fig. 4 Effect of nozzle gap on temperature. H_2-O_2 mixture; stoichiometric weight ratio; nozzle pressure, 80 psia.



same conditions as the all-brass configuration. The variation in temperature as a function of nozzle gap for both the brass and vacuum jacketed configurations is shown in Fig. 4. Ignition was achieved with the vacuum-jacketed configuration at a nozzle gap of $\frac{19}{64}$ in. The ignition and resulting combustion resulted in the melting of the vacuum jacketed apparatus.

While the results clearly demonstrate the feasibility of the resonance effect as an ignition method, there are several problems which must be solved before actual testing of an igniter in a rocket engine can be attempted.

The apparatus must be constructed of a material with low thermal conductivity to minimize the heat leak so that ignition can be obtained as well as a material which can withstand the intense heat of combustion following ignition. The resonance igniter must be able to operate over a range of ambient pressures and with propellant mixtures at cryogenic temperatures so as to be usable for engine restarts in space. The effects of the other variables must be studied to determine the optimum igniter configuration.

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Storage of Human Organs for Transplantation

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A METHOD for indefinite preservation of organs is critically needed. Such a method, when found, will permit establishment of organ banks. The limited supply of healthy organs from cadavers will be efficiently harvested by removal of several organs from each cadaver. Immunologic rejection will be minimized by preoperative tissue testing of the proposed recipient with multiple potential donors for the best compatibility. The timing of operation to fit the need of the recipient, and the use of only a single surgical team will enlarge the number of individuals receiving needed transplants.

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Short-Term Storage Techniques

The integrity and function of the cells of the human body are dependent upon complex intracellular chemical reactions. Interruption of this steady state of metabolism, whether by toxins, deprivation of nutrients, or accumulation of waste products results in cellular damage and eventual death. Various ingenious storage techniques have been devised to support cellular metabolism.

Perfusion with blood substitutes, combined with oxygenation and chemical replenishment, is practical for only a few hours. Irreversible swelling (edema) of the organ associated with circulatory stasis supervene. Perfusion by temporary vascular anastomoses to an intermediate host animal technically is feasible. Unfortunately, the immunologic rejection phenomenon prevents prolonged storage. Hypothermia to 0–5°C slows cellular metabolism reversibly for up to 90 min for whole organs. Storage is further prolonged under hyperbaric oxygenation at 3–7 atm combined with hypothermia.

Animal kidney, heart, and intestine have been preserved up to four days using hypothermia and hyperbaric oxygenation supplemented by a slow perfusion. The most impressive clinical results are those using a sophisticated hypothermic perfusion of oxygenated cryoprecipitated human plasma. Human kidneys have been preserved for as long as 32 hr prior to transplantation.[1] None of these techniques permits prolonged oxygen storage without deterioration as required for banking.

Frozen Storage of Cells and Blood

Although recovery of viable cells after freezing was first reported in 1949, commercially profitable processes for frozen storage of spermatozoa and bacteria are now in use. Human red blood cells stored for weeks by freezing, have been used in Viet Nam for transfusions.

Freezing and thawing processes are not innocuous to living cells.[2] Damage is related to the formation of ice crystals, the denaturation of proteins by dehydration, the changes in pH as buffer salts crystallize, the hypertonicity of preservative chemicals and the edema that accompanies cellular damage. Empirically, the use of water-binding substances such as dimethylsulfoxide (DMSO) and glycerol, along with precise temperature control have proven to be protective in certain situations (see Table 1). These chemicals are hypertonic in protective concentration, and are known to cause some degree of cellular damage. If cooling is too slow, if storage temperature is too high, or if heating is not extremely rapid, then the deterioration of tissues is great. Overheating beyond 42°C causes rapid destruction of biological tissues.

Table 1 Protection against freezing injury

Cryoprotective chemical	DMSO (Dimethylsulfoxide) or Glycerol
Rapid freezing	1–10°C/min
Cold storage	−196°C
Rapid thawing	25–100°C/min

Table 2 Ratio of surface area to volume

	(cm^{-1})
Red blood cell	16,000
Skin graft	30
Small intestine	8
Heart	2
Kidney	1
Adult human body	0.2